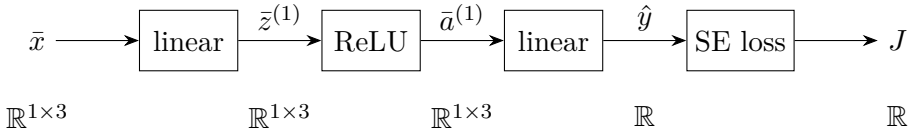
Quiz 3: Neural Networks (part 2) — Answer Key

E190BO *Instructor copy*



(a) Forward prop.

$$\bar{z}^{(1)} = \bar{x}W^{(1)} + \bar{b}^{(1)} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$$

$$\bar{a}^{(1)} = \text{ReLU}(\bar{z}^{(1)}) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$$\hat{y} = \bar{a}^{(1)}W^{(2)} + b^{(2)} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + 0 = 2$$

$$J = (y - \hat{y})^2 = (3 - 2)^2 = 1$$

(b) Backprop.

$$\frac{\partial J}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} \left\{ (y - \hat{y})^2 \right\} = -2(y - \hat{y}) = -2(3 - 2) = -2$$

$$\frac{\partial J}{\partial \bar{a}^{(1)}} = \frac{\partial J}{\partial \hat{y}} W^{(2)T} = -2 \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 2 \end{bmatrix}$$

$$\frac{\partial J}{\partial \bar{z}^{(1)}} = \begin{bmatrix} -2 & -2 & 0 \end{bmatrix} \qquad \leftarrow \text{only keep elements where } \bar{z}^{(1)} > 0$$

$$\frac{\partial J}{\partial W^{(1)}} = \bar{x}^T \frac{\partial J}{\partial \bar{z}^{(1)}} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} -2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 0 \\ -2 & -2 & 0 \\ 2 & 2 & 0 \end{bmatrix}$$

$$\frac{\partial J}{\partial \bar{b}^{(1)}} = \frac{\partial J}{\partial \bar{z}^{(1)}} = \begin{bmatrix} -2 & -2 & 0 \end{bmatrix}$$